'Are You Prepared?'

Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. The distance d from $P_1 = (3, -4)$ to $P_2 = (-2, 1)$ is $d =$ _____. (p. 5)

2. To complete the square of $x^2 + 5x$, add _____. (p. 991)

3. Find the intercepts of the equation $y^2 = 9 + 4x^2$. (pp. 15–17)

4. *True or False:* The equation $y^2 = 9 + x^2$ is symmetric with respect to the x-axis, the y-axis, and the origin. (pp. 17–19)

5. To graph $y = (x - 5)^3 - 4$, shift the graph of $y = x^3$ to the (left/right) _____ unit(s) and then (up/down) _____ unit(s). (pp. 118–120)

6. Find the vertical asymptotes, if any, and the horizontal or oblique asymptotes. if any, of $y = \dfrac{x^2 - 9}{x^2 - 4}$. (pp. 189–195)

Concepts and Vocabulary

7. A(n) _____ is the collection of points in the plane the difference of whose distances from two fixed points is a constant.

8. For a hyperbola, the foci lie on a line called the _____ _____.

9. The asymptotes of the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$ are _____ and _____.

10. *True or False:* The foci of a hyperbola lie on a line called the axis of symmetry.

11. *True or False:* Hyperbolas always have asymptotes.

12. *True or False:* A hyperbola will never intersect its transverse axis.

Skill Building

In Problems 13–16, the graph of a hyperbola is given. Match each graph to its equation.
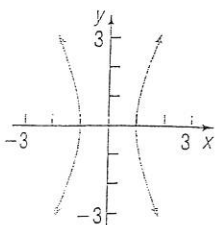
A. $\dfrac{x^2}{4} - y^2 = 1$
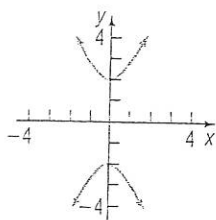
B. $x^2 - \dfrac{y^2}{4} = 1$

C. $\dfrac{y^2}{4} - x^2 = 1$
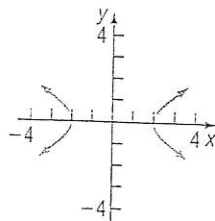
D. $y^2 - \dfrac{x^2}{4} = 1$
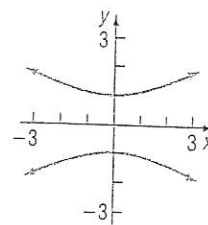
13.

14.

15.

16.

In Problems 17–26, find an equation for the hyperbola described. Graph the equation by hand.

17. Center at $(0, 0)$; focus at $(3, 0)$; vertex at $(1, 0)$

18. Center at $(0, 0)$; focus at $(0, 5)$; vertex at $(0, 3)$

19. Center at $(0, 0)$; focus at $(0, -6)$; vertex at $(0, 4)$

20. Center at $(0, 0)$; focus at $(-3, 0)$; vertex at $(2, 0)$

21. Foci at $(-5, 0)$ and $(5, 0)$; vertex at $(3, 0)$

22. Focus at $(0, 6)$; vertices at $(0, -2)$ and $(0, 2)$

23. Vertices at $(0, -6)$ and $(0, 6)$; asymptote the line $y = 2x$

24. Vertices at $(-4, 0)$ and $(4, 0)$; asymptote the line $y = 2x$

25. Foci at $(-4, 0)$ and $(4, 0)$; asymptote the line $y = -x$

26. Foci at $(0, -2)$ and $(0, 2)$; asymptote the line $y = -x$

In Problems 27–34, find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation (a) by hand and (b) by using a graphing utility.

27. $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$

28. $\dfrac{y^2}{16} - \dfrac{x^2}{4} = 1$

29. $4x^2 - y^2 = 16$
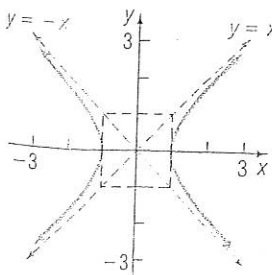
30. $4y^2 - x^2 = 16$

31. $y^2 - 9x^2 = 9$
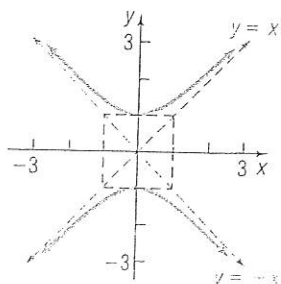
32. $x^2 - y^2 = 4$

33. $y^2 - x^2 = 25$

34. $2x^2 - y^2 = 4$

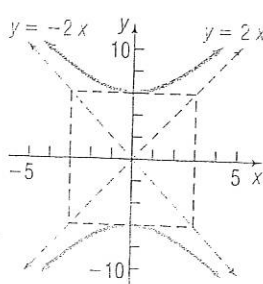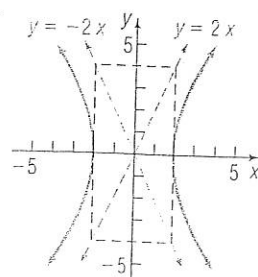*In Problems 35–38, write an equation for each hyperbola.*

35.

36.

37.

38.

In Problems 39–46, find an equation for the hyperbola described. Graph the equation by hand.

39. Center at $(4, -1)$; focus at $(7, -1)$; vertex at $(6, -1)$

40. Center at $(-3, 1)$; focus at $(-3, 6)$; vertex at $(-3, 4)$

41. Center at $(-3, -4)$; focus at $(-3, -8)$; vertex at $(-3, -2)$

42. Center at $(1, 4)$; focus at $(-2, 4)$; vertex at $(0, 4)$

43. Foci at $(3, 7)$ and $(7, 7)$; vertex at $(6, 7)$

44. Focus at $(-4, 0)$ vertices at $(-4, 4)$ and $(-4, 2)$

45. Vertices at $(-1, -1)$ and $(3, -1)$; asymptote the line $y + 1 = \frac{3}{2}(x - 1)$

46. Vertices at $(1, -3)$ and $(1, 1)$; asymptote the line $y + 1 = \frac{3}{2}(x - 1)$

In Problems 47–60, find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation (a) by hand and (b) by using a graphing utility.

47. $\dfrac{(x - 2)^2}{4} - \dfrac{(y + 3)^2}{9} = 1$

48. $\dfrac{(y + 3)^2}{4} - \dfrac{(x - 2)^2}{9} = 1$

49. $(y - 2)^2 - 4(x + 2)^2 = 4$

50. $(x + 4)^2 - 9(y - 3)^2 = 9$

51. $(x + 1)^2 - (y + 2)^2 = 4$

52. $(y - 3)^2 - (x + 2)^2 = 4$

53. $x^2 - y^2 - 2x - 2y - 1 = 0$

54. $y^2 - x^2 - 4y + 4x - 1 = 0$

55. $y^2 - 4x^2 - 4y - 8x - 4 = 0$

56. $2x^2 - y^2 + 4x + 4y - 4 = 0$

57. $4x^2 - y^2 - 24x - 4y + 16 = 0$

58. $2y^2 - x^2 + 2x + 8y + 3 = 0$

59. $y^2 - 4x^2 - 16x - 2y - 19 = 0$

60. $x^2 - 3y^2 + 8x - 6y + 4 = 0$

In Problems 61–64, graph each function.
[Hint: Notice that each function is half a hyperbola.]

61. $f(x) = \sqrt{16 + 4x^2}$

62. $f(x) = -\sqrt{9 + 9x^2}$

63. $f(x) = -\sqrt{-25 + x^2}$

64. $f(x) = \sqrt{-1 + x^2}$

Applications and Extensions

Fireworks Display Suppose that two people standing 2 miles apart both see the burst from a fireworks display. After a period of time, the first person standing at point A hears the burst. One second later, the second person standing at point B hears the burst. If the display is known to occur due north of the person at point A, where did the fireworks display occur?